



## Beethoven and Divergent Thinking

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### ABSTRACT

Works of art have value not only as entertainment and aesthetic expression – they also externalize features of human cognition that would often otherwise remain hidden. A musical example from the 19<sup>th</sup> century is a striking case in point. In 1819, Austrian music publisher, editor, and composer Anton Diabelli invited fifty-one of Austria’s most prominent musicians to submit one variation on a waltz theme he had created. Among those solicited was Ludwig van Beethoven. Though imperfect and no doubt unintentional, Diabelli’s project created a real-world experiment – fifty-one minds tackling the same creative problem. While every other composer submitted a single variation, Beethoven sent Diabelli thirty-three. Widely regarded as one of the greatest piano works of 19<sup>th</sup> century Europe, Beethoven’s *Diabelli Variations* represent a peerless display of divergent thinking. A cognitive investigation contrasting Beethoven’s approach with the practices of his contemporaries is highly revealing about the creative process, with implications for future research, educational practice, and artificial intelligence.

### ARTICLE HISTORY

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### Introduction


Works of art are often valued as entertainment and aesthetic expression, but they are more than that. Whether a monologue by Shakespeare, a self-portrait by Kahlo, a dance by Graham, or a fugue by Bach, they are a means of conducting research into ourselves – raising our awareness of the human mind at work (Martindale, 1990; Simonton, 2011). As researchers probe human cognition and study the workings of imagination, works of art offer a rich resource: they are a unique form of brain scan – externalizing features of human thinking that would otherwise often remain hidden (Brandt & Eagleman, 2017).

Thanks to composer and publisher Anton Diabelli, there is an unusually felicitous musical example. In 1819, Diabelli invited fifty-one of Austria’s most prominent musicians to each submit one variation on a waltz theme he had created (Roennfeldt, 2011). Among those solicited was Ludwig van Beethoven. While everyone else contributed one variation, Beethoven wrote thirty-three. Beethoven’s motives have never been firmly established. Some speculate that Diabelli offered him a large sum to compose an entire set. Others believe Beethoven just wanted to outdo his contemporaries. Whatever the reason, Diabelli published Beethoven’s set first in 1823, followed by the collection by the other fifty composers in 1824 (Roennfeldt, 2011).

Beethoven’s *Diabelli Variations* (1823) are widely acknowledged as one of the greatest piano works of 19<sup>th</sup>-century Europe (Cooper, 1985). The others, which Diabelli titled *Vaterländischer Künstlerverein*, (Brosche, 1983) have been largely forgotten. Beethoven’s set and Diabelli’s collection are rarely mentioned in the same breath, and few have thought it worthwhile to compare them. Though imperfect, Diabelli’s project unintentionally created a real-world creativity experiment: fifty-one minds tackling the same creative problem.

Investigating this robust group of professional musicians – among them, Franz Schubert and the 11-year-old Franz Liszt – makes this real-world scenario a nearly unprecedented case study. By comparing Beethoven’s variations with those of his contemporaries, this paper aims to add to existing literature analyzing works of art “using quantitative methods to test hypotheses concerning the creative process” (Weisberg, 2004, p. 23) – following in the footsteps of articles such as those by Weisberg (2004) and Simonton (2011) that draw different inferences from the sketches for Picasso’s *Guernica*. Viewed through a scientific lens, how Beethoven’s approach compares to his contemporaries’ is highly revealing about the creative process, with implications for future research, educational practice, and artificial intelligence.

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## How to compare variations

In a theme and variations, a musical statement is played over and over, but constantly in new ways. How to undertake a comparison? The most clear-cut way is to correlate the harmonic progressions: the choice and pacing of chords. This is for two reasons. First, whereas other aspects of musical analysis are harder to quantify, there is a well-established method – Roman numeral analysis – to label chords and their function. Second – and most importantly – in Beethoven’s day, the standard practice was for a theme and variations to maintain a strong grounding in its source’s harmonic progression. As the *Encyclopedia Britannica* explains, “A common feature of all variation types is the element of static structure, harmonically and tonally” (*Britannica*, 2014). A composer’s fantasy was expressed by transforming other musical features – motives, figuration, rhythmic patterns, and tempi. Meanwhile, the theme’s harmonic progression cycles over and over. Only in later variations might greater liberties be taken.

A set written by Archduke Rudolph (1992), written under Beethoven’s tutelage, is illustrative. The theme is by Beethoven (Figure 1).

Nineteen out of Rudolph’s first twenty-five variations map directly onto the theme’s harmonic progression (Figures 2-4).

Sets by Schubert, Czerny, Forster, Gelinek, Huttenbrenner, and Leidesdorf – all of whom answered Diabelli’s call – follow a similar paradigm.

Beethoven approached variation writing similarly. In his earliest set, *Nine Variations on a Theme by Drexler* (1782), the first eight follow the theme’s progression. Twenty years later, eight out of the first nine progressions in his “*Eroica*” *Variations*, Opus 35 (1802) match the theme in choice and pacing of chords. Having at least some of the variations match the source – especially initial ones – was Beethoven’s standard practice throughout his career. The majestic slow movement of the “*Archduke*” *Trio* (1811) is a characteristic example.

Thus, faithfully tracking the source progression – especially at the start and sometimes for the majority of variations – was a go-to strategy in this era. Fidelity to the source progression has a crucial consequence – the closer a variation’s progression is to the theme, the more recognizable the connection between them. Alter the progression more substantially, and that relationship becomes more tenuous. Comparing the composers’ approach to the harmonic progression is thus a strong indicator of how close or far they venture from the source – it is a measure of how harmonically adventurous they are.

## Answering Diabelli’s call

The analyses that follow aim to avoid technical jargon as much as possible. In Roman numeral analysis, chords are labeled by their position in the scale: I, ii, iii, IV, V, vi, vii°. Classical Western tonal progressions are based on a *tension-and-release* paradigm: the dominant chord, V, built on the 5<sup>th</sup> degree of the scale, represents motion and instability; it seeks to resolve to the tonic chord, I, built on the first degree, which represents rest, order, and the ultimate goal of motion. *Diatonic* progressions remain in the home key. *Chromatic* progressions, on the other hand, wander away, escalating the music’s unrest. Chromatic chords are labeled with respect to their target keys – for example, V/IV (read “V of IV”), V/V, and V/vi. *Cadences* are harmonic arrival points, articulating the ends of phrases. Finally, the *Major mode* is associated with positive emotions, and the *minor mode* with negative ones; changing modes provides expressive contrast. In Roman numeral analysis, Major chords and keys are marked in upper case and minor ones in lower.

Diabelli’s theme is divided into halves, each played twice, with the first half cadencing to V and the second to I. In the Figures that follow, significant departures from Diabelli’s source progression are marked in gray (in print) or red (online): even if you do not read music, you will be able to track these deviations.

Diabelli’s theme with harmonic analysis is shown in (Figure 5).

I            V            7            ii            vi            V            I            IV            vii°            I            V/V            V            7            I

Figure 1. Beethoven’s theme.

Figure 2 shows Rudolph's Variation 9, measures 1 through 6. The music is in 4/4 time and the key signature has one sharp (F#). The piano accompaniment consists of chords labeled I, V<sup>7</sup>, ii, vi, V, I, IV, V, I, V/V, V, 7, and I.

Figure 2. Rudolph's Variation 9.

Figure 3 shows Rudolph's Variation 14, measures 1 through 6. The music is in 12/8 time and the key signature has one sharp (F#). The piano accompaniment consists of chords labeled I, V<sup>7</sup>, ii, vi, V, I, IV, V<sup>7</sup>, I, V/V, V, and I.

Figure 3. Rudolph's Variation 14.

Figure 4 shows Rudolph's Variation 19, measures 1 through 4. The music is in 4/4 time and the key signature has one sharp (F#). A tempo marking indicates a quarter note equals 116. The piano accompaniment consists of chords labeled I, V, ii, vi, V, I, IV, V, I, V/V, V, and I.

Figure 4. Rudolph's Variation 19.

So how do the responses to Diabelli stack up? Out of the fifty contributions to the *Vaterländischer Künstlerverein*, four need to be

excluded as “free compositions:” Schenk’s *Capriccio*, Drechsler’s *Quasi Overture*, and S.R.D.’s fugue all pay homage to Diabelli’s theme but are

over 100 bars long, and Förster's *Capriccio* lasts almost 300. None of these can be matched directly with Diabelli's 36-bar progression.

That leaves forty-six variations to consider. Out of those, thirty-seven – approximately eighty percent – are faithful to Diabelli's progression or nearly so.<sup>1</sup> As such, we can classify them as “near” variations in close kinship with the theme. Put another way, four-fifths of the composers submitted variations characteristic of those occurring early in a set, when the affinity to the source tends to be strongest. The one by Conradin Kreutzer is a representative example (Figure 6).

Other examples include ones by Gottfried Rieger, J.P. Pixis, and Mozart's son, Wolfgang Jr. (Figures 7–9) Like Formula One racers, they are all driving the same course.

What about Beethoven? In his case, just the penultimate variation needs to be left out – it is an extended fugue. How many of the remaining thirty-two strictly adhere to Diabelli's choice and timing of chords? The answer? *None*. Beethoven's strategy is in sharp relief to the majority of his contemporaries. Throughout, he drives off-road, his variations at a greater displacement from their source. Beethoven certainly *could have*

covered more accessible ground: as noted, Diabelli was still awaiting submissions and published Beethoven's set first – so it wasn't a matter of the others' coming ahead of him. Yet not only did Beethoven never follow the most straight-forward solution, most of the other composers – at least some of whom were aware of what Beethoven had done – chose not to follow him into more rarified territory: for instance, Wittasek's, submitted last in 1824 (Roennfeldt, 2011), stays the course of Diabelli's progression.

There is more to it than that. Not only does Beethoven *never* copy Diabelli's progression, he also never repeats himself. No two variations follow the same progression. Kanwischer (2014) writes, “Although Beethoven honors his chorale as the established norm throughout, he deviates sooner, more often, more markedly – at last, dangerously” (p. 30). Yet while commentators have remarked on the particulars of individual variations, the evolution of their relationship to the theme, and the work's monumental scope (Geiringer, 1964; Kinderman, 1982; Cooper, 1985; Herzog, 1995), one is hard-pressed to find explicit acknowledgment that each variation is harmonically one-of-a-kind. No other composer of his era had ever attempted

Vivace



The musical score for Diabelli's theme is presented in four systems, each with a treble and bass staff. The tempo is marked 'Vivace'. The score includes various dynamics such as *p* (piano), *f* (forte), *sfz* (sforzando), and *ff* (fortissimo). Chord symbols are provided below the bass staff, including I, V<sup>7</sup>, V<sup>7</sup>/IV, IV, V<sup>7</sup>/IV, IV, V<sup>7</sup>/V, V, V<sup>7</sup>/vi, vi, V<sup>7</sup>/vi, vi, V<sup>7</sup>/vi, vi = ii/V, V/V, V, V<sup>7</sup>, I, V<sup>7</sup>/IV, IV, V<sup>7</sup>/IV, IV, V, V<sup>7</sup>/V, V, I, V, I, vii<sup>o</sup>, I, ii, V, I.

Figure 5. Diabelli's theme.

Vivace

The image displays a musical score for a piece titled 'Vivace'. It consists of five systems of music, each with a piano (right-hand) and bass clef (left-hand) staff. The score is annotated with various harmonic symbols and performance markings. The first system starts with a piano (*p*) dynamic and includes a first ending bracket labeled '1'. The second system features a series of chords labeled V/IV, IV, V/IV, IV, V/IV, V, V/IV, V, and V/vi. The third system includes a trill marked '8va 3' and a piano (*p*) dynamic, with chords labeled vi<sup>3</sup>, V/vi, vi<sup>3</sup>, V/vi, vi<sup>3</sup> = ii/V, V/V, V, V<sup>7</sup>, and V/IV. The fourth system has chords labeled IV, V/IV, IV, and V/IV. The fifth system has chords labeled V, V/V, V, I, V<sup>7</sup>, I, V<sup>7</sup>, I, ii, V, and I. The score concludes with a double bar line and repeat dots.

Figure 6. Kreutzer's Variation.

this before in a work of this scale. Perhaps it is only when set in relief against the *Künstlerverein* collection that the magnitude of what Beethoven has accomplished hits home – there is more harmonic variety *within* Beethoven's set than among the other forty-six composers combined.

Beethoven's restlessness impacts the variations themselves. The two halves of Diabelli's progression share an identical chromatic segment: twice  $V^7/IV$  going to IV, followed by twice  $V^7/V$  going to V. Only *three* out of thirty-two of Beethoven's variations maintain this parallelism. Variation X has a written-out repeat, with Beethoven reharmonizing each half: as a result, the chromatic segment is harmonized *four* different ways. Beethoven not only avoids duplicating himself from variation to variation, he also resists doing so *within* each variation. Given the opportunity to repeat himself, he consistently demurs.

How does he pull this off? Beethoven's harmonic transformations particularly take flight during the chromatic segments. Diabelli's chromaticism only references three keys: F-Major (IV), G-Major (V), and a-minor (vi) – all chords within to the tonic

key. In addition to those, Beethoven also references the diatonic keys of d-minor (ii) and e-minor (iii), along with more distantly related ones such as D-flat Major, E-flat Major, e-flat minor, f-minor, g-minor, A-flat Major, and B-flat Major. In revisiting the same key in different variations, he also scrambles the order and mixes the keys in different ways. There is a musical logic to Beethoven's strategy. The opening eight measures are more harmonically stable, making them more resistant to change. Beethoven only disrupts those approximately one out of four times. In contrast, the chromatic segment is more uprooted. As a result, he alters that portion of the progression three out of four times because there are many more options.

Amidst all of the transformations, how does Beethoven maintain audible connections to Diabelli's theme? The answer is that he dexterously keeps shifting which aspects of Diabelli's original to explicitly acknowledge – it might be the outline of the melody, a melodic or rhythmic motive, or a particular segment of the harmonic progression. There's always *some* nod toward the "influencer" – but what that *is* keeps changing. Beethoven's set would not exist without its

Vivace

8<sup>vo</sup>

1

V7

8

V7/IV IV V7/IV IV V7/IV V V7/IV V V7/vi vi V7/vi vi

14

V7/vi vi = ii/IV V7/V V V

22

1

V7/IV IV V7/IV IV V7/IV

28

V V7 I V7 I V7 I ii V7 I

Figure 7. Rieger's variation.

source, but only shards of the original are ever apparent. Consider the many ways Beethoven alludes to Diabelli's progression. For instance, Variation VIII opens appropriately by spending four bars on one harmony, then four bars on another—but this time, a substitute chord takes the place of V in mm 5–8 (Figure 10).

Meanwhile, the first eight measures of Variation XXVI begin routinely, but its subsequent four measures veer off unexpectedly (Figure 11).

And Variation V's first half concludes with a cadential progression – except that it's to the wrong key (Figure 12).

Even when Beethoven stays close to Diabelli's progression, notes outside it scar it like cracks in a façade (Figure 13).

On other occasions, just the scaffolding of the progression is all that is preserved: for instance, in the second half of Variation XXV, the progression is appropriately divided into four-bar units – but the harmonies don't line up with the original until the closing cadence (Figure 14).

Thus, while Beethoven's peers largely redecorate Diabelli's progression through figuration and texture, Beethoven sometimes tears Diabelli's progression all the way down to the studs – only its outlines remain.

Beethoven has a considerable advantage over his peers in writing an entire set. After all, if you are only submitting one, you are more likely to hew closely to the theme to avoid sounding uncooperative, unprofessional, or just plain weird. Indeed, the other fifty's contributions correspond to the theme in other ways – all but seven are in the original waltz's triple meter, and all but four are in Major. It is worth observing that among the few outliers are two of the most highly regarded contributors: the young Franz Liszt's variation is in duple meter and transfers Diabelli's progression to minor; and Schubert's is in minor as well, with a redesigned progression. Nevertheless, many other composers published their own full variation sets – including after Beethoven's death – and none ever attempted what Beethoven did in the *Diabelli Variations*. There is nothing wrong with what the other composers are doing. But

Vivace

The figure shows a musical score for a variation in 3/4 time, marked 'Vivace'. It consists of four systems of music, each with a treble and bass clef staff. Chord symbols are placed below the notes in the bass staff of each system. The symbols are: System 1: I, V<sup>7</sup>, V<sup>7</sup>/IV, IV, V<sup>7</sup>/IV, IV, V<sup>7</sup>/V; System 2: V, V<sup>7</sup>/IV, V, V<sup>7</sup>/vi, vi, V<sup>7</sup>/vi, vi, V<sup>7</sup>/vi, vi = ii/IV, V<sup>7</sup>/V, V; System 3: V<sup>7</sup>, V<sup>7</sup>/IV; System 4: IV, V<sup>7</sup>/IV, IV, V<sup>7</sup>/V, V, V<sup>7</sup>/V, V, I, V, I, V<sup>7</sup>, I, ii, V<sup>7</sup>, I.

Figure 8. Pixis' variation.

Beethoven is clearly innovating to a greater degree, with a lighter footprint of the source, an expansive proliferation of options, and the avoidance of duplication.

Though harder to quantify, there is another way in which Beethoven extrapolates more liberally from the source progression – his use of non-chord tones. Suppose you only had several outfits to wear. How could you keep them from becoming monotonous? One strategy would be to dress them up with different scarves, neckties, and jewelry. Non-chord tones fulfill a similar function in tonal music. By dexterously deploying non-chord tones, composers can generate an enormous variety out of tonality's basic sonorities. Beethoven uses non-chord tones extensively, uses different ones when referencing the same triad, and deploys them rhythmically to obscure a direct presentation of the harmony. For instance, Variation II stays closer than most to the source progression. However, non-chord tones shunt many of the harmonies to off-beats, accentuating the dissonance and chromaticism (Figure 15).

By varying the voicing, rhythmic placement, and choice of non-chord tones, Beethoven assures that recurrences of the same harmony rarely sound the

same way twice. All told, Beethoven's *Diabelli Variations* have remarkably little exact repetition compared to other music of his era.

Was Beethoven aware of what he was doing? Did he consciously decide *never* to follow Diabelli's progression? There is some circumstantial evidence. In reviewing Beethoven's early drafts, Kinderman (1987) notes that many of the variations originally overlapped and that

most of Beethoven's advanced work ... seems to have been directed towards imparting and perfecting an individuality of conception and sentiment. Often this process resulted in the de-emphasis, and even obliteration, of shared features (p. 38).

In the end, Beethoven produced a variation set that is harmonically non-recursive to an extent never before attempted in his day.

### Science and the "Diabelli Variations"

Divergent thinking – the generation of multiple solutions to an open-ended problem – is widely viewed as a crucial component of the creative process (Runco, 2010). Divergent thinking and creativity are not

*Con fuoco*



1 V<sup>7</sup>

7 V<sup>7</sup>

V<sup>7</sup>/IV IV V<sup>7</sup>/IV IV V<sup>7</sup>/V V V<sup>7</sup>/V V V<sup>7</sup>/vi vi V<sup>7</sup>/vi vi

14 V<sup>7</sup>

V<sup>7</sup>/vi vi = ii/V V/V V V<sup>7</sup>

20 V<sup>7</sup>

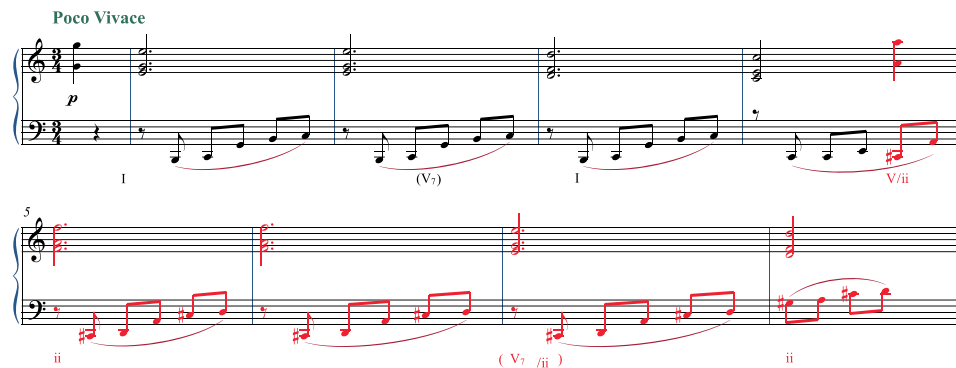
I V<sup>7</sup>/IV IV V<sup>7</sup>/IV IV V<sup>7</sup>/V

27 V<sup>7</sup>

V V<sup>7</sup>/V V V<sup>7</sup> I V I V<sup>7</sup> I ii V I

Figure 9. W.A. Mozart Jr.'s variation.

*Poco Vivace*



1 (V<sup>7</sup>)

5 V<sup>7</sup>/ii

ii (V<sup>7</sup> /ii) ii

Figure 10. Beethoven Variation VIII, mm 1–8.

synonymous: it is possible to think divergently without being very original (Runco & Acar, 2012). Nevertheless, it is hard to imagine arriving at novel solutions without a flourishing of options (Brandt & Eagleman, 2017).

A musical theme and variations is the epitome of a divergent thinking task (Brandt, 2019). Indeed, it might be nicknamed “Alternative Uses for a Theme,” aiming to generate musical statements of diverse personalities drawn from the same source.

The compositional challenges are comparable to divergent thinking tests (Silvia et al., 2008): create as many derivations as possible (*fluency*), make them as diverse as possible (*flexibility*), work them out carefully (*elaboration*), and contribute something new (*originality*). Notably, divergent thinking is not necessarily revealed in the final product: for instance, Hemingway wrote thirty-nine endings to *A Farewell to Arms* before choosing the one that





Figure 11. Beethoven Variation XXVI, mm 1–12.

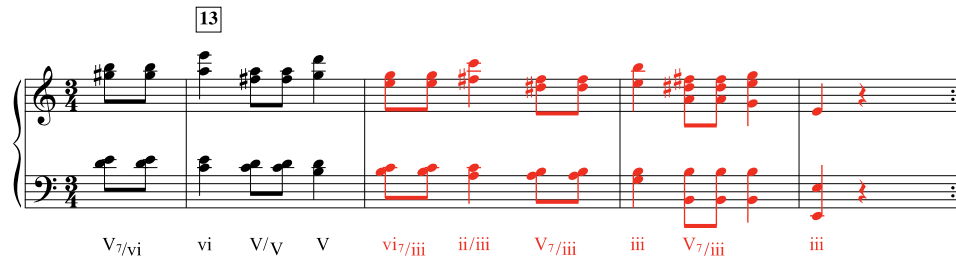


Figure 12. Beethoven Variation V, mm 13–16.

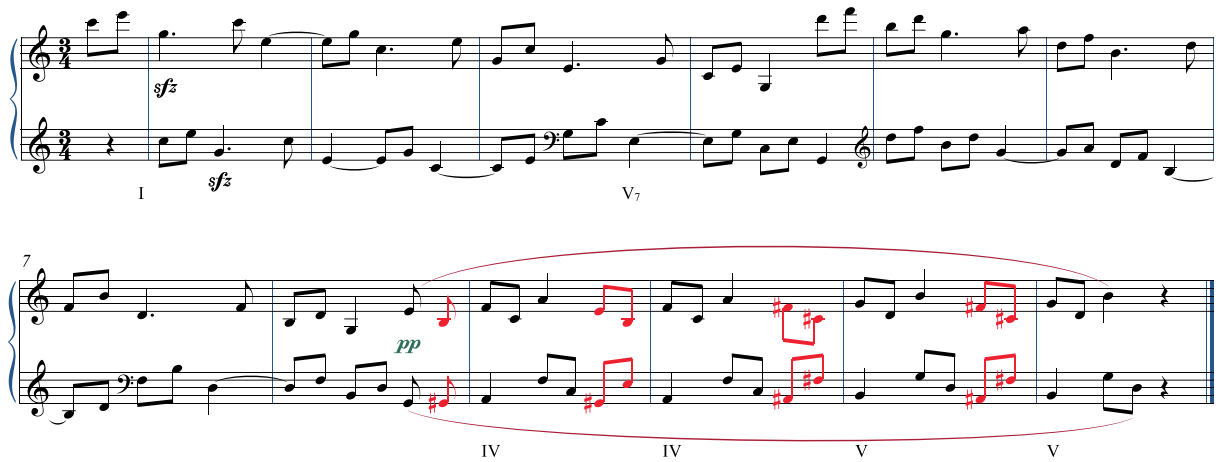


Figure 13. Beethoven Variation XIX, mm 1–16.

made it into print (Hemingway, Hemingway, & Hemingway, 2012). In contrast, theme and variations put divergent thinking on display, their proliferation of options shared with the audience.

Centuries before the first psychological inquiries, this musical form was one of human culture’s most explicit demonstrations of divergent thinking. Because of Beethoven’s unique approach, the

Figure 14. Beethoven, Variation XXV, mm 17–32.

*Diabelli Variations* represent a touchstone example.

### Beethoven and divergent thinking

The typical number of variations in a classical set ranges from six to ten. The highest number Mozart wrote in a stand-alone set is twelve. Among composers in general, there is a significant drop-off beyond that number. Beethoven's thirty-three thus rates at the high end for *fluency*. With thirty-two harmonically unique variants, Beethoven also scores at the extreme end for *flexibility*.

However, merely having a plentiful assortment of variations is insufficient. After all, some of these variations could be nonsensical or inchoate. Indeed, many of Beethoven's harmonic progressions are highly irregular: composer Arnold Schoenberg (1969) referred to this as Beethoven's most harmonically adventurous work. Yet, as generations of music theorists and musicologists have affirmed, Beethoven makes a convincing case for each one. In Kanwischer's words, Beethoven uses the "harmonic chorale of the theme as primary architectural

fundament, the manroot of a vast flowering" (Kanwischer, 2014, p. 31). As these experts attest, Beethoven earns top marks for *elaboration*.

Finally, *originality* is measured by statistical frequency – the rarer, the better. Thanks to Diabelli's call, we have a large pool of exemplars from which to refer. As noted, many in the general collection are harmonically alike. In contrast, when compared to other composers' submissions and each other, each one of Beethoven's solutions is unique. His set rates as original thirty-two times over.

Thus, not only are the *Diabelli Variations* great music, they are also an extravagant demonstration of divergent thinking. One could argue that, in part, what *makes* them great music is that they lay out so unequivocally the dynamics of human creative potential.

Experimental paradigms such as Alternative Uses and Alternative Consequences Tests are meant to be efficient proxies for the creative process (Hass & Beaty, 2018). However, they do not offer many clues about how ideas are generated. If, when asked, "What will happen if people no longer need to sleep?" a subject responds, "Coffee shops

**Poco Allegro**

The figure displays a musical score for Beethoven's Variation II, marked 'Poco Allegro'. It consists of four systems of music, each with a treble and bass clef staff. The score includes various chord annotations in red text below the bass staff, such as 'I', 'V', 'vii<sup>9</sup>/iv', 'IV', 'vii<sup>9</sup>/V', 'V', 'vii<sup>9</sup>/V', 'II<sup>6</sup>', 'V', 'vii<sup>9</sup>/vi', 'vi', 'vii<sup>9</sup>/vi', 'vi', 'vii<sup>9</sup>/vi', 'vi', 'V/v/V', 'vii<sup>9</sup>/V/V', 'V', 'V<sub>7</sub>/V', and 'V'. The music features a complex rhythmic pattern with many eighth and sixteenth notes, and some chromaticism.

Figure 15. Beethoven Variation II.

will close” or “Students will no longer skip 8 a.m. classes,” the appropriateness, variety, and uniqueness of their answers can be evaluated. However, it is difficult to understand how and why the ideas occurred to them. One advantage of a real-world task such as a musical theme and variations is that they provide a richer context for understanding the workings of the imagination. Beethoven places us by his side at his composing desk in the *Diabelli Variations*. We can hear up close and study the musical impetus for his distortions.

For instance, as we have seen, Beethoven achieves extraordinary diversity through his handling of Diabelli’s progression. In experiments asking participants to invent alien creatures, Ward, Patterson, and Sifonis (2004) found that those who were encouraged to treat a creative prompt more abstractly (considering environmental conditions and survival needs) rather than being tied to specifics (referencing actual Earth animals) produced more novel results. Similarly, rather than being bound to a literal reading of the progression, Beethoven approaches Diabelli’s blueprint schematically and is selective about what details to preserve. By meticulously varying which segments he leaves intact and which he modifies, Beethoven maximizes his proliferation of ideas.

### **Beethoven and the serial order effect**

The serial order effect asserts that the longer you spend on a divergent thinking task, the wilder and more unusual your ideas get (Beaty & Silvia, 2012). Although experiments tend to be highly time-limited – sometimes on the order of five to ten minutes – a meta-study by Paek, Abdulla Alabbasi, Acar, and Runco (2021) found that longer time spans significantly enhanced the originality of responses before eventually tailing off.

Because theme and variations sets consist of a sequential series of derivations from the same source, they offer a real-world way to examine this. The *Diabelli Variations* actually consist of two interpolated sets. Kinderman (1987) has established that Beethoven composed twenty-two of the variations in 1819; then, uncharacteristically, he set the work aside for several years – adding eleven more variations in 1823. How evident is the serial order effect within the first set and from the first to the second?

In the original set of 1819, the last six stand out as anomalous. No. 17, with its rhythmic stasis, unusual chromaticism, and nearly barren melodicism, is among the most unusual passages Beethoven ever wrote (it

occupies position XX in the final version). No. 18 very unusually splits each half into two different tempo markings. Kinderman reports that Beethoven labored over working this out. No. 19 is a parody, blending Diabelli's theme with Leporello's opening aria from Mozart's opera *Don Giovanni* – an incongruous pairing. No. 20 departs significantly from the source progression, its chromaticism taking it to a series of minor keys. Finally, no. 21 is itself in minor, and no. 22 is the fugue, which we've excluded from our comparisons.

As with the first set, the more unorthodox variations in the 1823 set come later, and, overall, this set is *more* divergent than the original one. When Beethoven interpolated the two sets, he placed most of his newer ones later in the sequence. Out of the last eleven variations in the complete set, eight of them date from 1823. As a result, “contrasts become more and more extreme” (Kanwischer, 2014, p. 64) as Beethoven gets deeper into his final order. Underscoring the *serial order effect*, the three variations before the penultimate fugue are all in minor – expressively far removed from Diabelli's jaunty waltz. Thus, Beethoven's variations trend toward the wilder and the stranger as the set unfolds. It supports the view that time is one of a creator's greatest allies – something not always apparent in short-term experiments (Amabile et al., 2002; Runco & Acar, 2012).

In reviewing the relevant seventy-eight responses to Diabelli's call, it is also worth noting that the closer variations are to the original, the more similar any alterations tend to be. For instance, many of the modest changes in the *Kunstlerverein* involve “spicing up the harmony” by substituting the more dissonant diminished 7<sup>th</sup> chord for a dominant – a chord to which it is closely related. Indeed, Beethoven's Variation II – one of his closest to the source – does the same thing. Meanwhile, the farther the variations get from the original, the less alike they are. For instance, the second half of his Variation XXII abruptly begins in the remote key of A-flat Major, while the first half sequence in Variation XXVIII is halved in length. No other variation by *any* of the composers does either.

Furthermore, none of the outliers in the *Kunstlerverein* match as well. For instance, Liszt and Schubert's variations are both in minor keys, but one tracks Diabelli's progression, and the other doesn't. Neither of them duplicates any of Beethoven's four variations in minor. *Opening up possibility space* is an apt way to describe what happens – variations closer to the theme tend to lie in similar orbits, while those farther away increasingly recede from each other. This illuminates a crucial feature of divergent thinking: when moving away from a source, there is no set path to follow.

## **Beethoven and domain knowledge**

Researchers have explored the interaction between domain knowledge and creativity. Some argue for the importance of solid grounding in a discipline as a prerequisite for creativity (Csikszentmihalyi, 2015); in an experiment by Rietzschel, Nijstad, and Stroebe (2006), participants primed in a task were judged as more creative than those who were naïve. Others have suggested that being overly concerned with domain knowledge can be constraining (Ward, 2008). The sweet spot between knowing too much and too little has been hard to pinpoint.

As a real-world example, what do Beethoven's *Diabelli Variations* tell us? As the composer of thirty-two piano sonatas and a host of variation sets, Beethoven's bona fides are well established – it is hard to argue that he was constrained by knowing too much. But there is another dimension not easily revealed by laboratory experiments – the longitudinal one. Nearing the end of his career, musicologists repeatedly point to the composer's desire to go beyond his earlier work (Cooper, 1985; Kinderman, 1987). All of Beethoven's late music makes a point of doing something he had never done before – he always accepted a degree of uncertainty. The premiere of his *Grosse Fuge* for string quartet is a case in point. Beethoven was so nervous about its reception that he could not bear to listen to the premiere in the hall. Instead, he waited in a bar across the street to hear how it went (Kahn, 2010). In the last decades of his life, Beethoven was nearly completely deaf – one of the only composers in Western history to continue his career despite losing his hearing. His escalated risk-taking in the *Diabelli Variations* is a testament to his musical command and desire not to revisit familiar territory. As with any explorer, Beethoven's combination of mastery and yearning for new vistas guided him into uncharted waters.

## **Sustaining, incremental, and breakthrough creativity**

One contentious issue in extant research is how to classify creativity. Kaufman and Beghetto's widely cited 4C model (2009) relies largely on impact and recognition. Novices (mini-c) and amateurs (little-c) conduct their work outside the public sphere, whereas professionals (pro-C) and those who achieve lasting fame (Big-C) share their work and garner professional approval. As the authors write, “Examples of Big-C creativity might

be winners of the Pulitzer Prize in fiction . . . or people who have entries in the Encyclopedia Britannica longer than 100 sentences” (2009, 2).

The dozens of variations submitted in response to Diabelli’s call suggest a complementary model: *sustaining*, *incremental*, and *breakthrough* creativity. *Sustaining* creativity is a “refresh” of an existing archetype, maintaining key features of the original. Its goal is not to replace the original but rather to perpetuate it. Renaissance ateliers, the 19<sup>th</sup> French Academie des Beaux-Arts, and movie sequels are signature instances of sustaining creativity. *Incremental* creativity seeks to upgrade an existing archetype. Annual updates to the iPhone and popular car models are examples. Here, the arrow of time matters more—you are unlikely to trade your newer computer for an older one. Finally, *breakthrough* creativity overturns precedent or so distorts it that it turns into something often unrecognizably novel – a more substantial break between before and after. This contrast is creativity at its most innovative.

You can belong to little-c, pro-C, and big-C and fall into any of these categories. For instance, Alfred Wegener, who proposed the theory of continental drift, and poet Emily Dickinson were viewed as amateurs in their lifetimes, yet both qualify as breakthrough creatives whose work was eventually acknowledged posthumously (Kirk, 2004; Schwarzbach, 1986). Meanwhile, Antonio Vivaldi, Felix Mendelssohn, Camille Saint-Saëns, and Sergei Rachmaninoff regularly appear on “Top 50” lists of classical composers (e.g., Pentreath, 2021; Pound, 2021). All four were extraordinarily imaginative and productive, yet none are generally regarded as great innovators. This observation is not meant as a slight. These composers were among the best of their time, and appraisals of their work remain high – they have earned their place among Western music’s canon. But they are known for elevating the mainstream, not overturning it.

The creativity displayed in the *Künstlerverein* collection consists mainly of the *sustaining* and *incremental* types. The variations are professionally written, and each contributes something distinctive – no two are alike. However, they remain faithful to established practice; and when they deviate from it, they don’t go far. Beethoven, on the other hand, through the sheer breadth of his variations, falls into the *breakthrough* category – he pours accelerants into the process of transformation.

Prioritizing *breakthroughs* over *sustaining* and *incremental* creativity may be a relatively recent phenomenon (Henrich, 2020). For instance, Elgammal and Saleh (2015) scored the originality of European religious paintings across several centuries in a digitized database. Originality scores were low in the 15<sup>th</sup> century, rose

throughout the Renaissance, and peaked in the 20<sup>th</sup> century. Likewise, Godin (2009) notes that neither Renaissance poets nor scientists were rewarded for their originality, and in the 16<sup>th</sup> and 17<sup>th</sup> centuries, “patents . . . were not granted to inventors, as they are today, but to importers of *existing* inventions, as a way to develop the local economy” (Godin, 2008, 12). Rapid overhaul and disruption may be Silicon Valley’s target, but it is a relatively recent objective.

Taking a modern attitude toward disruption is one reason why Beethoven is often viewed as ahead of his time. His intensity of remodeling precedent did not become a cultural lodestar until several generations later. Not only was he innovative – but being *that innovative* was itself an outlier within his musical culture. In that respect, Beethoven had few peers until the twentieth century, when composers such as Debussy, Scriabin, Ives, Schoenberg, and Stravinsky propelled a musical avant-garde that broke apart common Western practice into a proliferation of distinct musical languages. As you listen closely to Beethoven’s *Diabelli Variations*, you hear a prescient vision of where the modern world’s attitude toward creativity would be headed and a guide to how to think that way.

Researchers such as Csikszentmihalyi (1988), Glăveanu (2010), and Muthukrishna and Henrich (2016) have questioned the “lone genius” view of creativity. The latter write:

Just as thoughts are an emergent property of neurons firing in our neural networks, innovations arise as an emergent consequence of our species’ psychology applied within our societies and social networks. Our societies and social networks act as *collective brains* (2016, 2).

The *Diabelli Variations* offer an illuminating perspective. On the one hand, the German-born Beethoven moved to Vienna because it had a community and infrastructure that could support his life as a composer. He depended on his peers to write enough music to keep publishers in business and enough symphonies to keep orchestras employed. Furthermore, Diabelli’s call turned a typically solitary creative endeavor into a group activity. Lastly, it’s plausible that Beethoven’s prodigious number of variations may have been fueled at least in part by competitiveness – as a younger man, he participated in numerous improvisation duels (Blind, 1948). All of these embed Beethoven in his milieu.

On the other hand, Beethoven’s deafness led to his increasing isolation, intensifying his already manifest interest in experimentation (Solomon, 1998). Other than Schubert and Liszt, it is hard to support the idea that if Diabelli had reached out to any of the other fifty composers for a complete set of variations, they would have created

comparable work. A resolution to the “lone genius” vs. “collective brain” question undoubtedly lies halfway between. Without Western notation, harmonic progressions, the invention of the fortepiano, a vibrant community of fellow composers, etc., there would be no Beethoven. However, rather than faithfully passing on the DNA of his culture, Beethoven mutated it. The *Diabelli Variations* are a quintessential example of “culturally-impregnated’ materials” leading to “the generation of artifacts that are evaluated as new and significant” (Glăveanu, 2010) – the outcome of a collision between social interactions and personal initiative.

### **Beethoven and education**

“Make a copy and alter it” is a fundamental cognitive tool for producing novel output (Brandt & Eagleman, 2017). For instance, Picasso produced several hundred variations of Manet’s *Le Dejeuner sur l’herbe*. Architects routinely iterate designs for their buildings, and Apple has designers whose only job is to experiment with the shape of its packaging (Lashinsky, 2012). This intentional alteration is as true in STEM fields as in the arts. For instance, so-called “Intermediate Derivatization Methods” are used in pharmacology and agrochemicals to alter existing compounds in response to new toxins or pests (Berthon, 2020). As Sir James Black, winner of the 1988 Nobel Prize in Physiology and Medicine, said, “The most fruitful basis for the discovery of a new drug is to start with an old one” (Chen, Wu, Gao, Chen, & Zhou, 2016). Beethoven is “making a copy and altering it” thirty-two times over in his piano work, and what he is doing can be taught, practiced, and encouraged in the classroom (Brandt & Eagleman, 2017). The composer may set a high bar – but he also shows us how to jump.

In respecting Diabelli’s call, we cannot know what the other composers would have done if given the opportunity to submit more than one solution. But we *do* know that the composers who wrote one-offs tended to hew closely to their source. Meanwhile, the lone composer who created multiples produced the most original work. Consider an invitation like Diabelli’s being transferred to a classroom. Which would be more likely to drive the students toward more inventive solutions – one answer or many? If we want to train the next generation to be innovators, Beethoven’s *Diabelli Variations* offers a road map for doing so – they are as instructive as any textbook. The prompt is straightforward. Draw on an existing source and create as many derivations as possible – each distinct from the others. It is a call that can easily be repurposed across disciplines.

There is another lesson from Beethoven. Runco (2020) has written about creative *discretion* – knowing when to push the envelope and when to fall back on convention. Psychologist Frank Barron (1993) referred to this as “controllable oddness.” Beethoven is able to flex the variation schema without ever breaking it completely. All but two of his variations (XX, XXIX) are divided into halves. He keeps shifting what he preserves from the source progression but never abandons it entirely. And, despite numerous harmonic twists, every variation ends where it is supposed to – on the tonic. Thus, his departures can roam widely while still remaining on task. His disciplined flights of fantasy are a role model of how to be adventurous without losing one’s bearings.

### **Beethoven and AI**

The *Diabelli Variations* also have ramifications for AI creativity. For instance, Elgammal, Liu, Elhoseiny, and Mazzone (2017) have used adversarial networks to generate visual art. The networks consist of two algorithms – the *generator* and the *discriminator*. The *generator* – working without any preexisting data – creates images submitted for the *discriminator*’s approval. With each attempt, the *discriminator* draws on an extensive digital database of existing artwork to vote on whether the *generator*’s output qualifies as a painting and – if so – what style it is. A fitness function rewards the *generator* for increasingly meeting the *discriminator*’s standards. Without any knowledge base to work from, the *generator* begins randomly and, through repeated trials, gradually begins to produce images recognized as visual art by its “adversary.”

The goal is to develop an autonomous computational system that does not require human intervention to produce original artwork. Because Elgammal’s team is not interested in merely faithfully reproducing existing art, they add a twist: the *generator* is rewarded if the *discriminator* cannot confidently establish the style. In that way, the *generator* is encouraged to “surprise” the *discriminator* with an output it has never encountered before but still considers acceptable.

Using a similar strategy, the same call Diabelli issued in the nineteenth century could be extended to artificial intelligence. The *discriminator* could be provided with a database that approximates Beethoven’s knowledge base and programmed to compare variations to their source to understand the rubric. Meanwhile, the *generator*’s goal would be to compose music recognized as variations by the *discriminator*. As with visual art, we would expect the *generator* to begin naively and gradually produce

music that satisfies its automated judge. What tweaks to the algorithm could drive the *generator* into the creative territory of Beethoven's real-life *Diabelli Variations*? Could the computer be pushed to be as inventive, as resistant to self-duplication, yet in a way that still meets the *discriminator's* standards? Could it be trained to think like Beethoven?

To that end, there is evidence that creativity positively correlates with entropy: Shi et al. (2020) found that divergent thinking increased with less organized brain activation during resting states. In other words, creating something novel requires thinking in less predictable ways. Programmers could potentially incorporate measures of entropy into the algorithm. With low entropy, the *generator* would be rewarded for inventing variations closer to the consensus. With higher values, the *generator* would produce more unusual and unexpected output – yet still, satisfy the *discriminator*. There's no guarantee the algorithms would deliver “meaningful music:” artificial intelligence is unaware of music's expressive purposes or narrative structure. But dialing up the entropy until the *generator's* output approximates Beethoven's approach might yield insights into the creative process. Applying those insights to contemporary work could thereby help advance computer creativity.

Elgammal's team, advised by a group of musicologists, recently created a computer-generated realization of Beethoven's *Tenth Symphony* from sketches left by the composer at his death (Elgammal, 2021). The *Diabelli* challenge, however, offers some potential advantages. First, the *discriminator* requires massive data: Elgammal's data set for visual art included over 80,000 images. Fortunately, variations come in “bite-size” pieces, and sets from the nineteenth century are abundant. Because each set typically contains six to ten variations, a few dozen sets yields hundreds of exemplars – far more than symphonic literature can provide. Beethoven himself wrote nine symphonies but more than *sixty* sets of variations (Kanwischer, 2014).

Second, while he was alive, Beethoven was not trying to be Beethoven – he was trying to be *not*-Beethoven. As observed in the *Diabelli Variations*, he had an aversion to repeating himself. His later symphonies are a case in point. The Sixth is in five movements, the Seventh begins with a slow introduction, the Eighth lacks a slow movement, and the Ninth concludes with a flashback to earlier movements and adds a chorus. In his late music, even the length of works varies considerably. His *Piano Sonata No. 29, opus 106*, takes forty minutes to play; his next, *No. 30, opus 109*, takes half that time. In

trying to divine what Beethoven might have attempted in a Tenth Symphony, the best guess is he would cover new ground. But, as we have seen, the possibility space for a *breakthrough* creative is expansive, and it is impossible to forecast precisely which direction Beethoven would have headed. As a result, it is somewhat misleading to produce one realization of Beethoven's Tenth. It may satisfy experts that it sounds like Beethoven, but it cannot establish what Beethoven would have done. Algorithmically composing a full-length orchestral work is a major technical feat—but it is the computer's symphony, not Beethoven's.

As previously noted, a variation's goal is to broadcast a broad range of creative options. In contrast to symphonies – where exploring alternatives occur out of earshot in the composer's studio – variations put *divergent thinking* and *conceptual expansion* (Abraham et al., 2012) on display. In the case of the *Diabelli Variations*, Kinderman remarks on “how little of the material” in Beethoven's sketchbooks “goes unused . . . in the finished piece” (Kinderman, 1982, p. 29). Applying this to computational creativity better aligns with the open-ended nature of creative speculation, widens the scope of potential output, and allows for a more fine-grained analysis of how to leverage the algorithms to get closer and farther from the *discriminator's* database.

In Jorge-Luis Borges' story *Pierre Menard, Author of the Quixote* (Borges, 1981), a 20th-century writer tries to recreate Cervantes' *Don Quixote* word-for-word from his own lived experience without reading the original novel. Menard's fruitless quest succeeds in only producing a few scraps of text. Adversarial networks create artwork relatedly, with the *generator* naïve to precedent but striving gradually to approach it. We would not expect an algorithm to replicate Beethoven's *Diabelli Variation*. But we *could* evaluate whether it was *statistically* able to approximate the composer's radical approach. Variations potentially provide a sturdier test of algorithmic creativity than a realization of the Tenth Symphony—a composition for which no “authentic” original exists for comparison.

## Conclusion

Beethoven's *Diabelli Variations* represent a musical milestone. By elaborating upon a centuries-old musical form, Beethoven's work of peerless fluency and originality served as a message in a bottle for twentieth-century composers like Arnold Schoenberg, whose own notions of “perpetual variation” and lack of repetition profoundly impacted the Western avant-garde. Even today, theme and variations as a compositional form remain

a touchstone for composers. The set that most directly invokes Beethoven's *Diabelli Variations* is Frederic Rzewski's *The People United* (Rzewski, 1979), thirty-six variations on a Chilean protest song. Like Beethoven, Rzewski constantly remodels the harmonic progression, this time embracing a wide range of contemporary musical styles, including jazz and modernism.

The *Diabelli Variations* also represent one of the most overt real-world examples of divergent thinking on record. Creativity is best described as a tug-of-war between novelty and familiarity – too familiar, and we are likely to tune out; too novel, and we are likely to be confused (Brandt & Eagleman, 2017). Theme and variations are a playing field for this tug-of-war, with an audible connection to the theme competing with the challenges of refashioning it. The goal is to be both recognizable and new. Compared to the other contributors, eighty percent of whom hewed closely to Diabelli's harmonic progression, Beethoven was radically novel and non-repetitive, never following the most straightforward solution and never duplicating himself. Though Beethoven never entirely unties himself from the original, he loosens the strings far more than his peers.

The result is music that is constantly reinventing itself. Eagleman (2020) has described *pace layers* in the brain: we are more flexible and spontaneous in some aspects of our lives and more set in our ways in others. That distinction is reflected in how rigidly a given behavior is neurologically burned into our circuitry. While other composers of variation form had treated the surface features of the music as a divergent thinking problem, never before had anyone so thoroughly treated the underlying harmonic structure that way. In doing so, Beethoven excavated a pace layer that his peers often left undisturbed.

Acar and Runco (2019) describe a *cognitive hyperspace* where divergent thinking is represented along multiple pathways – original versus conventional, practical versus impractical, feasible versus unrealistic. In other words, divergent thinking is expressed on more than a single axis. Even in strictly musical terms, Beethoven's *Diabelli Variations* is a good fit for a multi-dimensional model, with the variations radiating out from a central focal point with respect to harmony, figuration, pacing, and mood. Rather than points on a line, widening orbits is a better way to visualize what happens. As Beethoven's variations recede from their source, they also recede from each other.

In an experiment underway at the University of Houston (UH Technology Bridge, 2021), visual artist Geraldina Interiano-Wise has been wearing a portable EEG cap that records her brain waves as she paints. Over time, an artificial intelligence program designed by neuro-engineer Pepe Contreras-Vidal and his team has analyzed

the EEG data and can predict Interiano-Wise's choice of color and brushstroke seconds before her physical movements. The computer's simulation is visible to Interiano-Wise on a monitor beside her easel, giving her the option to follow the computer's forecast or contradict it. If she conforms, the computer's model is reinforced; if she resists, it recalibrates. The experiment thereby makes visible an internal conversation that is often hard to document: an artist's conversation with her own history. In the *Diabelli Variations*, we see Beethoven figuratively wearing an EEG cap and never fully conforming to his past self.

It's impossible to know what other composers would have done if they had submitted entire sets or what Beethoven would have turned in if he had limited himself to one. Nevertheless, just as Simonton's *equal odds rule* (Simonton, 1997) puts a premium on production throughout a career, the results of Diabelli's call suggest that, when it comes to individual tasks, multiple solutions may offer more opportunities for adventurousness than solitary responses. While a single case study does not supply sufficient evidence to generalize or transfer results, deconstructing Beethoven's *Diabelli Variations* provides valid reasons for valuing the generation of multiple solutions over solitary responses. A person supplying a single solution will likely spend less time on the process. And, even if exploring a range of options, they might be more inclined to default to the familiar to avoid rejection. The originality of those tasked with providing individual vs. multiple solutions could be empirically tested with both amateurs and professionals incorporating a variety of tasks. If shown to be reliably predictive, the lesson for both the boardroom and classroom would be that encouraging a plethora of options from individuals and collaborators leads to more experimentation, innovative breakthroughs, and novel discoveries.

People tend to view artistic experiments as clearly distinct from scientific ones—artistic success measured by public acceptance and influence, while scientific success is measured by replicability and predictive power. Beethoven's *Diabelli Variations* show that artistic and scientific inquiry are more related than they might appear. In his thirty-three variations on Diabelli's theme, Beethoven not only broke new musical ground – he externalized basic cognitive features of human creativity. With supreme poetry and imagination, his music offers both aesthetic enjoyment and knowledge.

## Notes

1. Diabelli's progression turns the tonic (I) chord in m. 21 into a  $V^7/IV$  in m. 23 by adding a dissonant seventh. Going straight to  $V^7/IV$  in m. 21, as many composers



do, is a minor alteration; for all intensive purposes, the progression is the same.

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